that are not given by the tables; for example, the weight of water vapor when the relative humidity and the temperature are given, or the two temperatures of the psychrometer.

STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. Frank H. Bigelow.

VII.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT.

The data that have been collected regarding the meteorological conditions prevailing at the time of the Cottage City waterspout are sufficiently extensive and accurate to enable us to study carefully the causes that produced the phenomenon, and to derive several important results regarding the formation of lofty cumulo-nimbus clouds and the dynamic actions going on within them. In this instance we can compute approximately the forces producing the ascension of the buoyant vapor in the cloud, the formation of hail and the energy working in the vortex at the base of the cloud which developed as the waterspout. I shall proceed to give these facts in detail, as this computation may serve as a type to be followed in discussing other cases of similar local atmospheric action.

Besides the formation and dissipation of the tube, noted in the several reports and shown in figures 27–36, there are special features to which attention must be directed: (1) In the third appearance the vortex tube shows a gradual tapering of the form from the cloud to the sea level, but in the second appearance the tube seems to have about the same diameter from the cloud to the sea level. It is necessary to account for this divergence in the type. (2) The photographs show a peculiar set of boundary curves in the cloud level which depend upon certain dynamic forces that we shall attempt to discover. (3) At the foot of the tube, near the sea level, there was a great commotion of the waters, with a white nucleus just under the tube, and finally a beautiful cascade of imposing dimensions surrounding it. These are topics of especial interest besides those usually considered in discussing such vortices.

The meteorological conditions are given quite fully by the regular observations of the neighboring Weather Bureau stations, Nantucket being a station of the first order and having a continuous barograph and thermograph record; Woods Hole, a station of the second order, with complete daily evening observations; and Vineyard Haven, a station of the third order, with daily temperature, wind, and cloud reports. The daily weather map of 8 a. m., August 19, 1896, exhibits the general conditions for the United States, and from it can be obtained the local conditions prevailing at that hour, at least approximately. The physical appearance of the waterspout has been described fully in the reports already given, and there is also a series of notes of which further use will be made in the proper places. We shall endeavor in this Section, VII, to discuss the scientific problems which are naturally suggested by these data, with the view of illustrating typical methods of treating waterspout and tornado phenomena whenever these occur.

METEOROLOGICAL CONDITIONS FOR AUGUST 19, 1896.

Vineyard Haven, Marthas Vineyard, Mass.—The Journal for this station has been given as the report of W. W. Neifert, the observer, in the preceding Section, VI, page 307, extract A.

Nantucket, Mass.—The Journal for this station has been given as the report of Max Wagner, the observer, on page 309, extract C.

Woods Hole, Mass.—The report for this station has been given as the report of J. D. Blagden, the observer, on page 309, extract D.

This last report also adds:

Clear during the forenoon, partly cloudy during the afternoon. Thunderstorm: thunder first heard, 1:58 p. m.; loudest, 3:02 p. m.; last, 3:50 p. m. Storm came from the northwest and moved toward the southeast; temperature before the storm 66°, after 67°; direction of the wind before the storm northwest, after, northwest; during the storm the wind shifted to the northeast. Rain began 2:55 p. m.; ended 3:20 p. m.; amount 0:33 inch. Maximum wind velocity 38 miles per hour from the northwest, at 3:00 p. m. A few hailstones fell about 3:10 p. m., and quite a heavy fall of hail was reported a few miles north of this office.

The weather map of August 19, 1896, is represented as fig. 37.

In Section A of Table 50 the meteorological data are given for Woods Hole, Vineyard Haven, and Nantucket on August 19, 1896. They are extracted from Forms 1001–Met'l. of Woods Hole and Nantucket, and Form 1004–Met'l. of Vineyard Haven. The notation is as follows: B= barometric pressure; t=dry-bulb thermometer; t_1=wet-bulb thermometer; d=dew-point; R. H.=relative humidity; e=vapor tension; Max. t and Min. t=maximum temperature and minimum temperature for the periods ending at the respective times of observations; Dir.=direction and Vel.=velocity of the wind; Amt.=amount of precipitation; Amt.=amount; Kind; Dir.= direction from which the clouds came; Local time=hour of making the regular observations.

In Section B are given the meteorological data at the evening observation for ten days immediately preceding, and ten days immediately following the date August 19, 1896, with the purpose of showing the kind of August weather prevailing in that locality.

In Section C are given data at alternate hours obtained from the continuous self-register of the pressure and the temperature for Nantucket during August 19, 1896.

PROBABLE CONDITIONS NEAR THE WATERSPOUT.

The general chart for August 19, 1906, fig. 37, shows that an area of high pressure was central over the upper Lake region, with its eastern edge just overlapping the southern New England coast. This anticyclone was advancing quite rapidly for the summer season, and on the following day, August 20, it extended far eastward over the ocean. The winds were light to fresh from the north and northwest over New England, and the advancing border of the area of high pressure produced a showery condition with precipitation in eastern Maine, the upper St. Lawrence Valley, and on the Massachusetts coast. Later in the day several thunderstorms developed in the neighborhood of Vineyard Sound, such storms being reported as follows, seventy-fifth meridian time:

Station.	Began.	Loudest.	Ended.	Precipitation.
Woods Hole	1:58 p. m.	3:02 p. m.	3:50 p. m.	2:55 to 3:20 p. m.; 0.33 inch.
Vineyard Haven	1:45 p. m.	3:04 p. m.	3:45 p. m.	3:04 to 3:30 p. m.; 0.38 inch.
Nantucket*				2:40 to 4:00 p. m.; 0.03 inch.

*_Note by observer at Nantucket, Mass._—An ordinary thunderstorm was already passing across the sound, when about 12:40 p. m. a huge black tongue shot down from an alto-cumulus cloud that floated half a mile high at the northern edge of the shower.

Several observers mention the thunderstorm that occurred in the neighborhood, in connection with the formation of the waterspout at the base of a large cumulo-nimbus cloud. The day had been generally calm, the breeze being about three to six miles per hour from the northwest. There was a pronounced turbulent congestion of the atmosphere along the southeastern edge of the anticyclone, also a strong convectional movement in the vertical direction, as was indicated by the rapid formation of clouds, the production of precipitation, and the generation of thunderstorms accompanied by an overturning of the strata. All these are consequences attending the flow of cold anticyclonic air over the ocean, causing abnormal temperature stratifications to be superposed upon the ordinary quiet arrangement due to solar insolation taken by itself. The readjustment of the thermal equilibrium, which had become much congested, gave rise to the phenomena

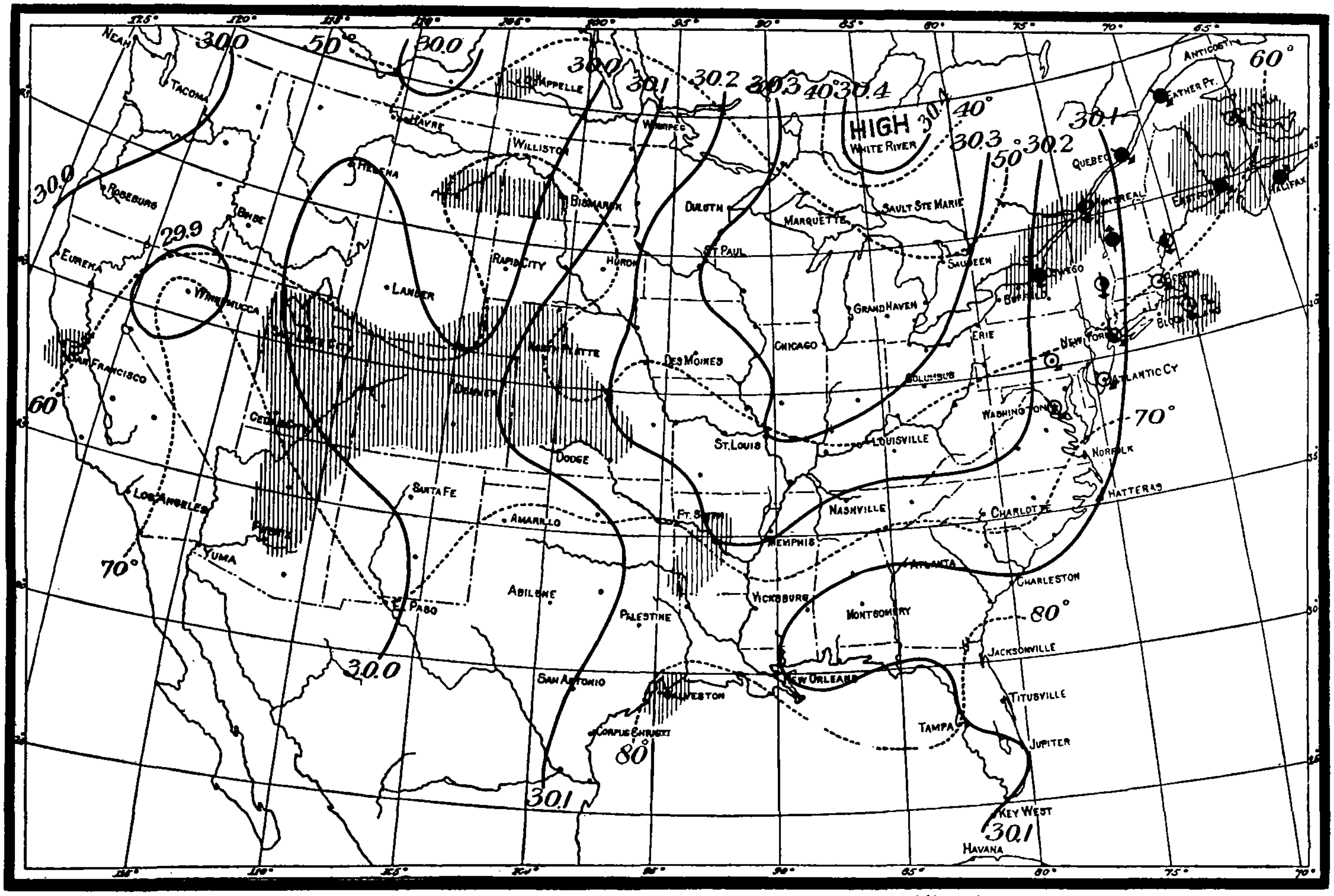

FIG. 37.—Weather conditions, Wednesday, August 19, 1896, at 8 a. m., seventy-fifth meridian time, preceding the waterspout.

observed on the occasion, one of which was the formation of the waterspouts from the base of the same thunderstorm cloud. The vertical convection producing the cloud became vigorous enough to generate vortex tubes during the interval, 12:45 to 1:28 p. m. Similar violent disturbances of the lower atmosphere occur in summer whenever masses of air of very different densities are brought close together, with abrupt changes in the temperatures within short distances. In the Mississippi Valley this usually occurs in the southeast quadrant of a cyclone, where the two streams flow together or overflow one another, one cool and dry from the northwest, the other warm and moist from the south. Near the Atlantic coast the same effect is produced by an anticyclonic area with dry, cool northwest winds pushing forward and protruding the current, flowing freely in the higher levels, over the warm, moist layers of air lying near the surface of the ocean. In all such cases we have a natural thermodynamic engine, where the pressure, volume, temperature (p_1, v_1, T_1) for the "source" of the thermal energy is quite different from (p_0, v_0, T_0) for the "sink", the former corresponding with the boiler and the latter with the condenser of an engine. Thus, (p_1, v_1, T_1) apply to the meteorological conditions in the warm air over the ocean before the anticyclone disturbs it, and (p_0, v_0, T_0) to the conditions prevailing in the cool air of the anticyclone itself. The motions of the atmosphere, which result in clouds, thunderstorms, and waterspouts simply represent the stream lines through which the air flows in restoring the abnormal temperatures to equilibrium. In nature there is a tendency to produce a succession of such thermal engines throughout the atmosphere, first on a large scale due to solar radiation, with the boiler along the Tropics, and the condenser around the poles; second on a small scale, wherever in the local circulations, which are secondaries derived from the first kind, masses of air of different temperatures come into close juxtaposition. Similar processes continue from step to step, from the general circulation on the entire hemisphere of the earth, to the smallest whirl that occurs in the turbulent internal vortex motions of the air.

Were it possible to trace the course of these stream lines throughout such congested masses as are in motion in tornadoes and thunderstorms, we should be able to study many interesting problems in hydrodynamics as well as in thermodynamics.

Examining the meteorological data I make the following deductions. *The temperature at the base of the waterspout was about 67.5°.* This is the maximum temperature of the day, and the thermograph for Nantucket shows that the waterspout occurred at the time of maximum temperature, just before the break in weather occurred. It then fell to about 56.5° at Vineyard Haven, and 59.0° at Woods Hole and Nantucket, so *that the effective temperature in the anticyclone was about 58.0°.* Hence we shall assume $t_1 = 67.5°$ F. and $t_0 = 58.0°$ F. *The pressure in the anticyclone may be taken 30.10, and on the ocean near the waterspout 30.05, which is only a little lower.* The table makes the relative humidity range above 90 per cent at Nantucket for several days till August 18. By the morning of August 19 a decided change had occurred, in which the relative humidity fell nearly to 60 per cent. At Nantucket it was 60 per cent at 8:20 a. m., and 74 per cent at 8:20 p. m.; at Woods Hole it was 61 per cent at 8:17 p. m. It is very remarkable that this great waterspout was *produced on the day when the lower strata of the atmosphere were drier than on any other*

(A) TABLE 50.—*Meteorological data for August 19, 1896.*

Station.	Pressure.	Temperature and moisture.							Wind.		Precipitation. Amt.	Clouds.			Local time of observation.
	B_0.	t.	t_1.	d.	R. H.	e.	Max. t.	Min. t.	Dir.	Vel.		Amt.	Kind.	Dir.	
	Inches.	° *F.*	° *F.*	° *F.*	*Per cent.*	*Inches.*	° *F.*	° *F.*		*Miles p. h.*	*Inch.*				
Woods Hole							64.0	59.0							8:17 a. m.
	30.15	64.0	56.2	50.0	61	.360	67.4	64.0	nw.	14	0.33	0	0	0	8:17 p. m.
Vineyard Haven							72.0	56.5	nw.		0.38				8:17 p. m.
Nantucket	30.03	64.0	56.5	51.0	62	.373	64.0	59.2	n.	10	0.02	few	Ci.	w.	8:20 a. m.
	30.13	62.0	57.0	53.0	74	.402	67.5	61.5	n.	8	0.03	0	0	0	8:20 p. m.

(B) *Records for 10 days either side of August 19, 1896.*

Dates.	Woods Hole, 8:17 p. m.							Nantucket, 8:20 p. m.						
	t.	t_1.	d.	R. H.	e.	Max. t.	Min. t.	t.	t_1.	d.	R. H.	e.	Max. t.	Min. t.
	° *F.*	° *F.*	° *F.*	*Per cent.*	*Inches.*	° *F.*	° *F.*	° *F.*	° *F.*	° *F.*	*Per cent.*	*Inches.*	° *F.*	° *F.*
August 9	73.0	71.8	72	94	.783	78.2	72.2	70.0	70.5	71	98	.757	78.0	70.2
August 10	76.0	71.5	69	80	.707	85.8	73.0	72.5	71.0	70	93	.732	86.3	72.5
August 11	76.7	72.5	70	82	.732	87.0	75.0	74.5	72.5	72	91	.783	83.2	74.5
August 12	75.2	73.0	72	90	.783	85.1	75.2	71.0	70.0	70	95	.732	81.8	71.0
August 13	72.5	71.5	71	95	.757	81.9	72.5	71.0	70.0	70	95	.732	82.0	71.0
August 14	66.5	66.0	66	98	.638	71.0	66.5	66.0	66.0	66	100	.638	74.0	66.0
August 15	65.2	64.0	63	94	.575	69.0	65.0	64.0	63.5	63	97	.575	67.5	64.0
August 16	69.2	67.0	66	89	.638	71.0	65.9	65.0	64.5	64	97	.595	66.8	63.2
August 17	68.8	63.8	61	76	.536	77.7	68.5	66.5	64.5	64	90	.595	74.2	66.5
August 18	62.2	58.2	55	79	.432	69.8	60.3	64.0	63.5	63	97	.575	73 0	64.0
August 19	**64.0**	**56.2**	**50**	**61**	**.360**	**67.4**	**64.0**	**62.0**	**57.0**	**53**	**74**	**.402**	**67.5**	**61.5**
August 20	64.8	59.0	55	71	.432	70.0	59.1	59.5	55.5	52	78	.387	69.0	59.0
August 21	65.0	58.3	54	67	.417	71.0	64.5	57.5	56.5	56	94	.448	68.5	57.5
August 22	67.0	61.0	57	71	.465	71.8	65.9	64.5	60.0	57	77	.465	71.0	64.5
August 23	71.0	70.2	70	96	.732	73.3	68.2	69.0	68.0	68	95	.684	74.6	66.1
August 24	71.5	69.0	68	88	.684	76.1	69.5	67.5	65.5	64	90	.595	75.5	67.5
August 25	65.0	62.8	62	89	.555	75.7	64.0	64.5	64.0	64	97	.595	74.8	64.0
August 26	68.0	65.0	63	85	.575	73.2	66.3	64.0	62.5	62	92	.555	73.8	63.5
August 27	68.0	65.0	63	85	.375	73.8	67.0	68.5	67.0	66	93	.638	74.0	63.3
August 28	65.3	58.8	54	68	.417	70.7	60.8	61.5	56.5	52	74	.387	68.0	61.5
August 29	64.0	58.0	54	70	.417	68.0	62.9	58.0	56.0	55	89	.432	68.8	58.0

(C) *Continuous self-register of pressure and temperature at Nantucket, Mass., August 19, 1896.*

Hours of seventy-fifth meridian time.	Mid't.	2 a. m.	4 a. m.	6 a. m.	8 a. m.	10 a. m.	Noon.	2 p. m.	4 p. m.	6 p. m.	8 p. m.	10 p. m.	Mid't.
Pressure, in inches	29.99	29.97	29.96	30.00	30.03	30.05	**30.05**	**30.05**	30.07	30.09	30.12	30.15	30.16
Temperature, degrees Fahrenheit	61.0	60.0	61.0	62.5	64.0	65.0	**66.0**	**67.0**	63.0	64.0	63.0	63.5	63.0

day of that month. A comparatively low humidity was prevailing during the formation of the vortex, and even the thunderstorms of the afternoon did not avail to increase it much, as it rose only to 74 per cent in the evening. There was no doubt considerable local fluctuation in the humidity for short intervals, but the prevailing anticyclone lowered the humidity for the duration of two or three days. We have therefore the special problem of accounting for the waterspout during conditions which were quite the reverse of those usually assumed to prevail. Tornadoes are usually found connected with warm, moist air, but here we find cool, dry air, showing that it is not high surface temperature and humidity alone that causes these vortices. After several trial computations on the cloud dimensions, which depend upon the correct surface data, and allowing a small rise in the humidity percentage from 8:17 a. m. to 1 p. m. over the 61 or 62 per cent prevailing at the early hour, I have concluded to accept R. H.=64 per cent as that prevailing near the surface of the water at the time the waterspout was formed. We therefore have to begin the computations with the following data for the air just above the surface of the water at 1 p. m.:

$$B = 30.05 \text{ inches.}$$
$$t = 67.5° \text{ F.}$$
$$\text{R. H.} = 64 \text{ per cent.}$$

COMPUTATION OF THE PRESSURE B, TEMPERATURE t, VAPOR-TENSION e, AND HEIGHT H, FOR THE α, β, γ, δ STAGES.

We now have the meteorological data at sea level beneath the cloud which surmounted the waterspout, as shown in the photograph (fig. 29, 2d C), taken at about 1:08 p. m., by Mr. E. K. Hallet. This cloud is a large cumulo-nimbus; its flat base is about 3500 feet above the sea level, and its apex about three miles high. Upon the southern extension, on the right-hand side of the plate, there is a thunderstorm and the rain is falling freely; the northern side is clearing, and there seems to be sunshine in several places; from the midst of the cloud the waterspout projects down to the sea level. At the top of the waterspout tube the curvature of the vortex is well defined, and at the bottom there is a fountain of water surrounding it in a circular cascade. We shall determine as accurately as possible the meteorological elements (B, t, e) at the base of the cloud, that is, at the top of the a-stage, which extends from the sea level to the lower face of the cloud, and expresses the fact that this stratum of air is composed of dry air and invisible aqueous vapor mixed in certain proportions, as indicated by the relative humidity, which is 64 per cent at sea level and 100 per cent at the base of the cloud. We shall use the notation:

h_1, B_1, t_1, e_1 at the top of the a-stage, or the base of the cloud.
h, B, t, e at the bottom of the a-stage, or the sea level.

Compare the notation on page 677 of the International Cloud Report.

(1) The working formula for the a-stage is No. 145, as given on page 496 of the same cloud report:

$$C_\alpha = \text{Constant} = \left(0.2374 + 0.1512\,\frac{e}{B} + 0.0232\,\frac{e^2}{B^2}\right)\log T. \quad \mathrm{I}_\alpha$$

$$(145) \qquad - \left(0.06858 + 0.02592\,\frac{e}{B}\right)\log B \ldots\ldots\ldots \mathrm{II}_\alpha$$

where T is the absolute temperature $= 273^\circ + t^\circ$ centigrade. This formula is reduced to the numerical Tables 94, 95, 96, pages 550 to 553 of that report, inclusive, the T and B terms being summarized by $C_\alpha = \mathrm{I}_\alpha + \mathrm{II}_\alpha$. Some examples of the use of these tables are given in that report on page 573 and following, also page 765 and following, the accompanying text having the necessary precepts for practical work. They will be further illustrated in the examples offered by this waterspout.

(2) The β-stage extends from the base of a cloud to the plane of freezing temperature, $t = 0°$C., which in this cumulus cloud lies at about 2800 meters, or 9200 feet, above sea level, so that the β-stage stratum is not far from 1700 meters or 5600 feet thick. In the present case the α-stage is about two-thirds of a mile deep, and the β-stage a little more than one mile deep. The β-stage consists of a mixture of dry air, invisible aqueous vapor, and condensed aqueous vapor in the form of minute water drops or cloud particles which reflect and refract the light, and thus make the cloud appear as a visible mass or a region of condensation. The formula for the β-stage is No. 146 as given on page 496:

$$C_\beta = \text{Constant} = \left(0.2374 + 0.4743\,\frac{e}{B} + 0.145\,\frac{e^2}{B^2}\right)\log T_1 \ldots \mathrm{I}_\beta$$

$$(146) \qquad - \left(0.06858 - 0.04266\,\frac{e}{B}\right)\log (B_1 - e_1) \ldots \mathrm{II}_\beta$$

$$+ \left[\frac{497.5}{T_1} - 0.4404 - \left(\frac{309.4}{T_1} - 0.2739\right)\frac{e}{B}\right]\frac{e_1}{B_1 - e_1} \ldots \mathrm{III}_\beta\,,$$

where the notation is taken from Table 133, page 677, of the Cloud Report. The numerical tables are given on pages 554 to 556 and Tables 97, 98, and 99 for the successive terms, I_β, II_β, III_β, and examples of the computation may be found on pages 574, 696 and following, also in this paper.

(3) The γ-stage stratum is shallow, in this case only 75 meters or 246 feet thick, and it contains the layer of atmosphere within which the water of the cloud is turning to ice, the temperature remaining constant at the freezing point during this process. In this layer the aqueous content is changing its physical state, though not its temperature, and there is only a mixture of aqueous vapor, water, and ice, since there is no thermal change in the dry air because of the constant temperature, the cooling by expansion just balancing the warming due to evolution of latent heat.

This is a very interesting process and will be referred to again in a later section of this paper. The formula is No. 147, on page 496, in the notation of the Cloud Report, page 677:

$$C_\gamma = \text{Constant} = -\left(0.06858 - 0.04266\,\frac{e}{B}\right)\log (B_0 - e_0) \ldots\ldots \mathrm{II}_\gamma$$

$$(147) \quad + \left[\frac{547.2}{T_0} - 0.4404 - \left(\frac{340.4}{T_0} - 0.2739\right)\frac{e}{B}\right]\frac{e_0}{B_0 - e_0} \ldots \mathrm{III}_\gamma$$

$$-\left(49.78\,\frac{e}{B} + 18.82\,\frac{e^2}{B^2}\right)\frac{1}{T_0} \ldots\ldots\ldots\ldots\ldots, \quad \mathrm{IV}_\gamma$$

The table for II_γ is the same as II_β, page 555, and those for III_γ and IV_γ are given on page 557. Numerical examples can be found on page 575, also on page 697 and following; likewise compare the γ-stage of the Cottage City waterspout, later in this paper, under (c).

(4) The δ-stage relates to a stratum of mixed dry air, frozen aqueous vapor, i. e., ice vapor, and ice or snow below the freezing temperature, and in the present case this stratum extends from the height of about 2880 meters, or 9450 feet, above the sea level to the apex of the cloud which is not far from 4940 meters, or 16,200 feet; so that the δ-stage, or the snow-stage, is about 2060 meters, or 6750 feet in depth, that is about one and one-fourth miles thick. The formula is No. 148 on page 496, in notation of pages 677 or 678, as follows:

$$C_\delta = \text{Constant} = \left(0.2374 + 0.4743\,\frac{e}{B} + 0.145\,\frac{e^2}{B^2}\right)\log T_{11} \ldots \quad \mathrm{I}_\delta$$

$$(148) \qquad -\left(0.06858 - 0.04266\,\frac{e}{B}\right)\log (B_{11} - e_{11}) \ldots \quad \mathrm{II}_\delta$$

$$+ \left[\frac{547.2}{T_{11}} - 0.4404 - \left(\frac{340.4}{T_{11}} - 0.2739\right)\frac{e}{B}\right]\frac{e_{11}}{B_{11} - e_{11}} \ldots \mathrm{III}_\delta$$

The numerical tables are on pages 555, 558, and 559, the table for II_δ being identical with that for II_β. Examples of the frozen or δ-stage may be found on pages 576, 697–712, also in the following computations on this waterspout. It is convenient to have a distinct notation for the bottom and top of each of these four stages, and this is done in the notation of pages 677 and 678 of the Cloud Report by simply transferring the suffix of the bottom of the stage to the place of an exponent at the top of that stage. The notation for the top of one stage is equivalent to that of the bottom of the next higher stage, and both symbols may be employed.

In the computation of h, B, t, e for the several stages of the Cottage City waterspout explicit directions will now be given for each stage in the work, for the sake of providing suitable precepts to be followed by others pursuing this line of research, and in order that the several steps may be clearly and accurately understood. The tables for each stage, pages 550 to 559, were all computed so that the absolute temperature, $T = 273° + t°$C., which appears in the formulæ is found as $t°$ C. in the argument of the tables, for the sake of convenience in computing.

(A) THE α-STAGE OR UNSATURATED PROCESS.

The sea-level conditions as above given may be converted into the metric system by the following table:

	English system.	Transferred to the metric system.	Metric system.
B	30.05 in.	By Smithsonian Table 64	763.27 mm.
t	67.5° F.	By Smithsonian Table 2	19.72° C.
Relative humidity	64 per ct.		64 per cent.
e		By Smithsonian Table 43, e for saturation at 19.72°, Broch's value, is 17.06 mm. and 64 per cent of this is—	10.92 mm.
$\frac{e}{B}$		$10.92 \div 763.27 =$	0.0143
Reduction of $\frac{e}{B}$		From Table 136 p. 690, Cloud Report.	—0.0013
Corrected $\frac{e}{B}$		$0.0143 - 0.0013 =$	0.0130

The result of the discussion of the value of the ratio $\frac{e}{B}$ at the sea level and at the base of the cumulus cloud, as given on pages 677 to 693 of the Cloud Report, was to show that the ratio $\frac{e}{B}$ at the sea level in the α-stage has a larger value than at the base of the cloud. This is due to the fact that in nature the atmospheric process does not exactly follow the adopted theory of the α-stage, and is not a true adiabatic process as the formula requires. We can, however, adapt the formula to this case by applying to the sea-level value of $\frac{e}{B}$ a slight correction to make it smaller at the base of the cloud, and, as the outcome of a very extensive discussion of this problem, the necessary corrections are given in Table 136, page 690, of the Cloud Report for metric measures, and on Table 18, page 107, of the Barometry Report for English measures. It is necessary to determine this ratio quite accurately for close computations of the humidity correction, according to the theory adopted in these two reports. In this connection the reader is referred to further remarks on this formula, which has been in use since 1897 by the Weather Bureau, as found on page 781 of Dr. J. Hann's Lehrbuch der Meteorologie, 1901, and on page 30 of the Meteorologische Zeitschrift of January, 1903, by Dr. J. Liznar, where the formulæ are proved to be sufficiently accurate for all practical computations. The Table 136 gives for the argument $\frac{e}{B} = .0143$ and the assumed approximate height of the base of the cloud, 1100 meters, a correction which I take as $-.0013$, so that we have $.0143 - .0013 = .0130$ as the value of $\frac{e}{B}$ to be employed throughout the computation, at least so long as precipitation does not produce pseudo-adiabatic conditions. While there is rain falling from the cloud at some distance to the south, where the approximate adiabatic process has been disturbed, it yet seems proper to retain the full ratio $\frac{e}{B} = 0.0130$ for computing along the line from the waterspout to the apex of the cloud, since it is probable that the activity of the cloud continues still to be nearly adiabatic along the main line of ascension of the vapor of condensation. It is to be particularly noted that this ratio, $\frac{e}{B} = 0.0130$, is to be used in computing every stage, and is the quantity that controls the value of the pressure, temperature, and vapor tension at all the levels of the cloud. The fact that the heights computed by the thermodynamic processes are found to be in close agreement with those measured on the photograph, according to the data of the survey, also tends to justify the adoption of that value of the ratio $\frac{e}{B}$.

The constant C_a in the a-stage is computed as follows:

I_a	1.3596	Table 95, arguments $\left(t = 19.72, \frac{e}{B} = 0.0130\right)$.
II_a	$-$ .4574	Table 96, arguments $\left(B = 763.3, \frac{e}{B} = 0.0130\right)$.
C_a	0.9022	Sum of I_a and II_a = constant.

These values should be determined with accuracy to the fourth decimal place throughout the computation, and the result will then be sufficiently exact to follow the natural processes exactly as they develop. The significance of the constant $C_a = 0.9022$ can be understood from this consideration. We have a set of mutually related thermodynamic quantities, $B = 763.3$, $t = 19.72°$, $e = 10.92$, $\frac{e}{B} = 0.0130$, connected together in such a way as to give the constant 0.9022.

There are many other sets of values of B, t, e, which can be developed by changing the thermodynamic system adiabatically, that is to say without adding or subtracting any heat. Now, as a given mass of the aqueous vapor, so many grams per thousand grams of air, or so many thousandths of one kilogram, rises from the sea level and ascends upward to the base of the cloud, it evidently passes through pressures and temperatures which are each diminishing in amount. Then at a certain height, where the pressure and temperature are sufficiently reduced, this same mass of vapor will begin to saturate the kilogram, and condense as visible vapor. In all the steps of change in B, t, e from the surface to the cloud-base, the same constant $C_a = 0.9022$ must be retained, and also the same ratio $\frac{e}{B}$ must be kept, so that there remain the two variables B and t. Now the connection between t and e is fixed at a certain value by the saturation tables, so that if t_1 *is assumed*, e_1 is found by the table, and B_1 is obtained by computing backwards through Tables 95, 96, the result being checked by having the ratio $\frac{e_1}{B_1} = 0.0130$. This usually requires a few trials to effect correctly, and one needs a little practise with the data to do it expeditiously. In order to make a beginning with the trial temperatures it is well to remember that in dealing with cumulus clouds the average fall in temperature in passing from the sea level to the base of the cloud, through 1100 meters, is usually 10° to 11° C. This is shown in Table 147, where for the surface temperature $t = 20°$, and $\frac{e}{B} = 0.0143$, which is the original value of $\frac{e}{B_1}$, we have for the temperature gradient per 1000 meters, $Gt = -9.50$. This table is adapted for the cumulus cloud-base at the height of 1000 to 1100 meters as it stands, without modification. If the temperature is for any other height, the gradient must be modified somewhat as indicated by the subtable on page 727, though the gradients may become very abnormal under certain conditions. It is proper from these considerations to begin the trial values of t_1, the temperature at the base of the cloud, with $t_1 = 9°$, and $t_1 = 10°$. These values will give resulting values of the $\frac{e_1}{B_1}$ which will enable us by interpolation to find the correct value of t_1 at the third trial.

The following form illustrates the method of procedure for assumed successive values of t_1. The last column gives the arguments, or the method of obtaining the figures in the second and third columns:

t_1	9°	10°	(Assumed.)
e_1	8.55	9.14	Smithsonian Table 43.
I_a	1.3507	1.3515	Table 95, arguments. $\left\{ t = 9°, \frac{e}{B} = 0.0130;\ t = 10°, \frac{e}{B} = 0.0130 \right.$
C_a	.9022	.9022	From preceding result, formula 145.
II_a	$-$ .4485	$-$ .4493	$C_a - \mathrm{I}_a = \mathrm{II}_a$.
B_1	670	678	Table 96, argument. $\frac{e}{B} = 0.0130$.
$\frac{e_1}{B_1}$	0.0128	0.0135	Resulting ratio of $\frac{e_1}{B_1}$.

This ratio should have come out $\frac{e_1}{B_1} = 0.0130$ if the trial values of t_1 had been correct, and interpolation shows that in order to make it so the value of t_1 should be 9.3°.

Repeat the computation,

t_1	9.3°	
e_1	8.72	Smithsonian Table 43.
I_a	1.3509	Cloud Report Table 95.
C_a	.9022	As above.
II_a	$-$ .4487	
B_1	672	Cloud Report Table 96.
$\frac{e_1}{B_1}$	0.0130.	This completes the check, and gives $B_1 = 672$, $t_1 = 9.3°$, $e_1 = 8.72$ at the base of the cloud.

We next compute the height at which the visible cloud base floats above the sea level.

For Table 91 we have the formula of barometric reductions and the corresponding height,

$$\log B_0 - \log B_1 = +m - \beta m - \gamma m.$$

		Logarithm.
B_0	763.27	2.88268
B_1	672	2.82737
$m-\beta m-\gamma m$		.05531

Since Table 91 is constructed to find m through the arguments, height $= H$ and mean temperature $T_m = \theta$, in order to obtain H by the inverse process we must compute m itself from $m-\beta m-\gamma m = .05531$, by taking $m = .05531 + \beta m + \gamma m$.

By Table 92, with the arguments ($B_0 = 763$ and $e_0 = 10.9$) we have:

For argument $\mathrm{I} = .378 \frac{e_0}{B_0} = \ldots\ldots\ldots$ .0054 and
For argument $\mathrm{II} = \beta$ (for assumed $H = 1100$) .0048
So that $+\beta m$ (for $m = .055$) .... $+0.00026$
By Table 93, $+\gamma m$ (for $\varphi = 41.5°$) .... $+0.00001$
Hence, since $\log B_0 - \log B_1 =$ 0.05531

We have, m 0.05558

Finally the temperature $\theta = \frac{1}{2}(19.7 + 9.3) = 14.5°$.

By Table 91, with the arguments ($\theta = 14.5$ and $m = 0.05558$) we find by interpolating $H_a = \begin{cases} 1078 \text{ meters.} \\ 3537 \text{ feet.} \end{cases}$

We thus find, from the thermodynamic computation, that the height from the sea level to the plane of condensation at the base of the cloud is 3537 feet. From the measurements on fig. 27, 2d A, as contained on page 311, the approximate height is 60 mm. or 3600 feet. Considering the uncertainty in determining the exact plane which is the base of the cloud sheet, where the vortex motion becomes asymptotic, it is evident that we have reached a satisfactory agreement regarding the height by two independent methods. Furthermore, this check is a good evidence that the fundamental data in these computations are in harmony geometrically and thermodynamically, and we may, therefore, feel confident that the other deductions depending upon them are substantially correct.

The gradients per 100 meters for the a-stage are now easily obtained.

	mm.		°C.			
B_0	763.3	t_0	19.7	e_0	10.92	whence $H_a =$
B_1	672.0	t_1	9.3	e_1	8.72	1078 meters.
$B_1 - B_0$	-91.3	$t_1 - t_0$	-10.4	$e_1 - e_0$	-2.20	

Divide $B_1 - B_0$ etc. by 10.78, or the height in units of 100 meters.

$(G.\,B)_O - 8.46$ $(G.t)_O - 0.963$ $(G.e)_O - 0.204$.. By observation.
$(G.\,B)_C - 8.40$ $(G.t)_C - 0.950$ $(G.e)_C - 0.192$.. Table 147, I, II, III Cloud Report.

The tabular gradients are found in Table 147, page 724, which gives the gradients that prevail for average conditions, by using the arguments $\left(t_0 = 19.7 \text{ and } \frac{e}{B} = 0.0145\right)$. The agreement is such as to show that the meteorological conditions forming the waterspout cloud are entirely in harmony with those which are found to prevail under *similar circumstances*, when thunderstorms and tornadoes are in action. If, however, we compare them with the gradients found under the normal August conditions as given in the Barometry Report for Nantucket, we have the following data taken from page 707:

	Inch.		°F.		Inch.	
B_m	29.977	t_0	67.7	e_0	.571	English
B_1	26.470	t_1	60.4	e_1	.445	measures.

Convert these into metric measures.

	mm.		°C.		mm.	
B_m	761.35	t_0	19.83	e_0	14.50	Metric
B_1	672.34	t_1	15.78	e_1	11.30	measures.
$B_1 - B_m$	-89.01	$t_1 - t_0$	-4.05	$e_1 - e_0$	-3.20	

Divide $B_1 - B_m$ etc. by 10.78, or the height in units of 100 meters.

$(G.\,B)_B$ -8.24 $(G.\,t)_B$ -0.375 $(G.\,e)_B$ -0.296

The discussion of these gradients will be resumed, as they are important in the theory of atmospheric vortices.

(B) THE β-STAGE, OR SATURATED PROCESS.

The computation of the numerical value of the meterological elements (B^1, t^1, e^1) at the top of the β-stage, or the bottom of the γ-stage (B_0, t_0, e_0), proceeds as follows:

t_1	9.3° C.	Taken from the top of the a-stage.
B_1	672 mm.	
e_1	8.72 mm.	
$\frac{e}{B}$	0.0130	The same as adopted for the a-stage.
$B_1 - e_1$	663.28	
$\frac{e_1}{B_1 - e_1}$	0.0132	

		Arguments.
I_β	$+1.3747$	Table 97 $\left(t = 9.3°, \frac{e}{B} = .0130\right)$.
II_β	$-\ .4419$	Table 98 $\left(B_1 - e_1 = 663.3, \frac{e}{B} = .0130\right)$.
III_β	$+\ .0173$	Table 99 $\left(t = 9.3°, \frac{e_1}{B_1 - e_1} = .0132\right)$.
C_β	0.9501	Constant for the β-stage.

At the top of the β-stage two quantities are fixed, namely, temperature $t^1 = 0°$ and $e^1 = 4.57$ mm., so that the term I_β can be at once found and subtracted from the constant.

t^1	0°	
e^1	4.57 mm.	
I_β	1.3664	Table 97, arguments $\left(t = 0°, \frac{e}{B} = .0130\right)$.
C_β	0.9501	C_β taken from above.
	-0.4163	Subtract I_β from C_β.

The term -0.4163 is made up of II_β and III_β at the top of the β-stage, and these depend upon finding by trials B^1, the pressure there prevailing. It is convenient to employ Hertz' well-known diagram of adiabatic changes in the $a\ \beta\ \gamma\ \delta$ stages, a copy of which is given in Professor Abbe's Translations, The Mechanics of the Earth's Atmosphere, page 202, in order to have approximate values to figure the trial computations. Enter the β-stage at the point $B_1 = 672$, $t_1 = 9.3°$, and follow the system of β lines up to $t = 0°$ and $B = 552$. Make two trials by two assumptions of the pressure B^1.

B^1	552.0	542.0	Assumed values of B^1
e^1	4.57	4.57	
$B^1 - e^1$	547.4	537.4	
$\frac{e^1}{B^1 - e^1}$	.0084	.0085	
II_β	$-.4289$	$-.4276$	Table 98, arguments $\begin{cases} B^1 - e^1 = 547.4, \frac{e}{B} = .0130 \\ \text{“} \quad 537.4, \quad \text{“} \end{cases}$
III_β	$+.0115$	$+.0116$	Table 99, arguments $\begin{cases} t = 0, \frac{e^1}{B^1 - e^1} = .0084 \\ \text{“} \quad \text{“} \quad = .0085 \end{cases}$
II+III	$-.4174$	$-.4160$	

The term II+III should be equal to $-.4163$, and hence we interpolate another assumed value of $B^1 = 544$. Repeat the trial,

B^1	544	Assumed by interpolation.
B^1-e^1	539.4	
$\frac{e^1}{B^1-e^1}$	.0085	

Arguments.

II_β —.4279 Table 98, $\left(B^1-e^1=539.4, \frac{e}{B}=.0130\right)$

III_β +.0116 Table 99, $\left(t=0, \frac{e^1}{B^1-e^1}=.0085\right)$

II+III —.4163 The check is complete for $B^1=544$.

The computation for H_β, the depth of the β-stage, is as follows:

		Logarithms.			
B_1	672	2.82737	$e_1=8.72$	$t_1=9.3°$	$\theta=4.7°$
B^1	544	2.73560	$e^1=4.57$	$t^1=0.$	

			Arguments.	
$m-\beta m-\gamma m$	0.09177			
βm	+ 38	Table 92	I =.0050,	II=.0041
γm	+ 1	Table 93	$e_1=8.7$	$H=1700$
			$B_1=672$	$m=.092$
m	0.09216			

$H_\beta=$ { 1728 meters. Table 91, arguments ($\theta=4.7°, m=.09216$). 5669 feet.

The gradients for the β-stage are now found,

	mm.		°C.		mm.	
B^1	544	t^1	0		$e^1=4.57$	$H_\beta=1728$ meters.
B_1	672	t_1	9.3		$e_1=8.72$	
B^1-B_1	— 128	t^1-t_1	—9.3		e^1-e_1 —4.15	

Divide B^1-B_1 etc. by 17.28, or the height in units of 100 meters.

$(G.B_1)_O$ —7.40 $(G.t_1)_O$ —.538 $(G.e_1)_O$ —.240 By observation.
$(G.B_1)_C$ —7.40 $(G.t_1)_C$ —.540 $(G.e_1)_C$ —.260 Table 147, IV, for $\frac{e_1}{B_1}=.0130$.

In order to compare this with the normal conditions of the atmosphere in August, we again take the Nantucket data, Barometry Report, page 707, where (B, t, e) are given on the 3500-foot plane and the 10000-foot plane.

The depth of the α-stage $= H_\alpha = 1078$ meters $= 3537$ feet.
The depth of the β-stage $= H_\beta = 1728$ meters $= 5669$ feet.
Height of top of β-stage above sea level ...... H^1 2806 meters = 9206 feet.

By interpolation to the given heights, 1078 meters=3537 feet and 2806 meters=9206 feet, we obtain in metric measures, from page 707 of the Barometry Report, the data.

	mm.		°C.		mm.
B^1	548.6	t^1	9.22	e^1	4.95
B_1	671.6	t_1	15.72	e_1	11.25
B^1-B_1	— 123.0	t^1-t_1	— 6.50	e^1-e_1	— 6.30

Divide by 17.28, the height of the β-stage in units of 100 meters.

$(G.B_1)_B$ — 7.11 $(G.t_1)_B$ — .376 $(G.e_1)_B$ — .364

These sets of gradients for the β-stage will also require further discussion.

(C) THE γ-STAGE, OR FREEZING PROCESS.

In the freezing stage there is no change in the temperature, which is $t_0=0°$, nor in the vapor tension, which is $e_0=4.57$ mm., and we have only to compute the variation in the pressure B_0.

Constant $\mathrm{II}_\beta+\mathrm{III}_\beta$	— .4163	brought from the β-stage.
t_0	0°	
B_0	544.	
e_0	4.57	
$\frac{e}{B}$	0.0130	

Assume

B^0	539.	for the top of the γ-stage.
e^0	4.57	
B^0-e^0	534.4	
$\frac{e^0}{B^0-e^0}$	.0086	

Arguments.

II_γ —.4273 Table 98 $\left(B^0-e^0=534.4, \frac{e}{B}=0.0130\right)$.

III_γ +.0134 Table 100 $\left(t=0, \frac{e^0}{B^0-e^0}=.0086\right)$.

IV_γ —.0024 Table 100 $\left(t=0, \frac{e}{B}=0.0130\right)$.

Sum —.4163 This satisfies the check.

For the depth of the γ-stage we compute,

		Logarithms.
B_0	544.	2.73560
B^0	539.	2.73159

$m-\beta m-\gamma m$ .00401 The corrections for $-\beta m-\gamma m$ can be neglected.

$H_\gamma=$ ... { 74 meters. Table 91, argument ($\theta=0°, m=.00401$) 243 feet.

The pressure gradient in the γ-stage is,

$(G.\ B_0)_O=6.76$ By observation.
$(G.\ B_0)_C=6.70$ Table 147, V, of the Cloud Report.

(D) THE δ-STAGE, OR FROZEN PROCESS.

The δ-stage extends to the visible tops of the clouds, and we may, therefore, take such a temperature as will, through the intermediate computation, produce a height which agrees with the height measured on the photograph. A preliminary trial of —11°C. for the temperature gives a height that is somewhat lower than the apex of the cloud, and a second trial of —12°C. seemed to be about right, so that it has been adopted for the numerical example.

For the constant in the δ-stage, we have,

t_{11}	0°C.	brought from the top of the γ-stage.
B_{11}	539.	
e_{11}	4.57	
$B_{11}-e_{11}$	534.4	
$\frac{e_{11}}{B_{11}-e_{11}}$	.0086	
$\frac{e}{B}$	.0130	

Arguments.

I_δ 1.3664 Table 101, $\left(t=0, \frac{e}{B}=.0130\right)$.

II_δ — .4273 Table 98, $\left(B_{11}-e_{11}=534.4, \frac{e}{B}=.0130\right)$.

III_δ + .0134 Table 102, $\left(t=0, \frac{e_{11}}{B_{11}-e_{11}}=.0086\right)$.

δ-Constant .9525

Assume t^{11} —12° C. Argument.

e^{11} 1.64 mm. Table 103, $t=-12°$C.

I_δ 1.3556 Table 101, $\left(t=-12, \frac{e}{B}=.0130\right)$.

Constant .9525

$\mathrm{II}_\delta+\mathrm{III}_\delta$ — .4031 This is the constant to determine B^{11}.

Assume B^{11}	415.0	419.0
e^{11}	1.6	1.6
$B^{11}-e^{11}$	413.4	417.4

$\dfrac{e^{11}}{B^{11}-e^{11}}$	.0040	.0040	
II_δ	$-.4098$	$-.4105$	Table 98, argument. $\left(B^{11}-e^{11}, \frac{e}{B}=.0130\right)$.
III_δ	$+.0066$	.0066	Table 102, arguments. $\left(t=-12°, \frac{e^{11}}{B^{11}-e^{11}}=.0040\right)$.
	$-.4032$	$-.4039$	

Interpolation indicates $B^{11}=414.5$.

Assume B^{11}	414.5		
e^{11}	1.64		
$B^{11}-e^{11}$	412.9		
$\dfrac{e^{11}}{B^{11}-e^{11}}$	.0040		Arguments.
II_δ	$-$ .4097	Table	$98\left(B^{11}-e^{11}=412.9, \frac{e}{B}=.0130\right)$.
III_δ	$+$ .0066	Table	$102\left(t=-12°, \frac{e^{11}}{B^{11}-e^{11}}=.0040\right)$.
	$-$ .4031	This checks the constant for the pressure $B^{11}=414.5$ mm.	

The depth of the δ-stage is found as follows:

		Logarithms.	Arguments.
B_{11}	539.0	2.73159	
B^{11}	414.5	2.61752	
$m-\beta m-\gamma m$		.11407	$\mathrm{I} = .0033\,(e=4.57, B=539)$.
$+\beta m+\gamma m$		.00029	$\mathrm{II} = .0026$ (assume $H=2000$).
m		.11436	$\beta m = .0026 \times .114 = .00029$.

$$\theta = \frac{0-12°}{2} = -6°$$

$$H_\delta = \ldots \begin{cases} 2062 \text{ meters.} & \text{Table 91 } (\theta=-6°,\ m=.11436). \\ 6765 \text{ feet} \end{cases}$$

(Arguments.)

The gradients in the δ-stage are now found:

	mm.		°C.		mm.	
B^{11}	414.5	t^{11}	-12.0	e^{11}	1.64	$H_\delta = 2062$ m.
B_{11}	539.0	t_{11}	00.0	e_{11}	4.57	
$B^{11}-B_{11}$	-124.5	$t^{11}-t_{11}$	-12.0	$e^{11}-e_{11}$	-2.93	

Divide $B^{11}-B_{11}$ etc. by 20.62, or the height in units of 100 meters.

$(G.\,B_{11})_o - 6.04$ $(G.\,t_{11})_o -.582$ $(G.\,e_{11})_o -.142$ By observation.
$(G.\,B_{11})_c - 6.50$ $(G.\,t_{11})_c -.550$ $(G.\,e_{11})_c -.140$ Table 147; VI, VII, Cloud Report.

The top of the cloud is about 5000 meters above sea level, so that in Section VII, Table 147, for $H=5000$ and $\frac{e_1}{B_1}=.0130$, we find the vapor tension gradient $(G.\,e_{11})_c = -0.140$ per 100 meters.

Table 51, summary of the data for the Cottage City waterspout, August 19, 1896, contains the results of the preceding computations on the thermodynamic conditions prevailing at that time, in both the metric and the English systems of measures. The first column of figures under each system gives the vertical distances H, measured in millimeters and inches on Hallet's photograph, 2d C, taken at about 1:08 p. m.; the second, the corresponding height in meters and feet; the third, the pressure in millimeters and inches; the fourth, the temperature in degrees centigrade and Fahrenheit; the fifth, the vapor tension in millimeters and inches; the last column the gradients as extracted from the Cloud Report from the same data; also the gradients as deduced from the Barometry Report for the same season of the year. By inspection, it is noted:

(1) In the α-stage in the English measures, the pressure gradient has increased from -0.098 to -0.101 per 100 feet. The normal gradient for August at Nantucket, only a few miles from the scene of the waterspout, is -0.098 per 100 feet, making a pressure fall of -3.47 inches from the sea level to the base of the cloud, or a change of pressure from 30.05 inches to 26.58 inches; the waterspout gradient -0.101 per 100 feet gives a fall of -3.58 inches and a pressure of 26.47 inches at the height 3537 feet, the base of the cloud. There is thus a total change in the vertical gradient of 0.11 inch from the sea level to the cloud, and it is this increased difference of pressure which causes the air to rise generally from the surface of the sea to the cloud, and is intimately concerned with the generation of the vortex tube. We may remark in passing that one of our purposes in constructing tables and charts of the normal values of the pressure, temperature, and vapor tension on the 3500-foot and the 10,000-foot planes, in connection with their values on the sea-level planes, was to afford the data for establishing the *normal vertical gradients* in the lower strata of the atmosphere, which shall represent the average stratification when undisturbed by convectional action. It is very desirable that these gradients should be checked by numerous direct observations of B, t, e, taken day and night, winter and summer, so that the normal gradients can be separated from the convectional gradients in a perfectly reliable manner. As this must be the work of years for the United States, it is permissible to employ the gradients contained in the Barometry Report, which rest upon the best data we now possess, in such discussions as are suggested by the Cottage City waterspout. Similarly, the phenomena of tornadoes in the Mississippi Valley, of thunderstorms generally, of cyclones, anticyclones, and hurricanes can be properly studied only by comparing the abnormal gradients, prevailing in these conditions of rapid convection, with these normal gradients. The differences between these two systems of gradients represent the energy which produces these atmospheric motions, and any explanation of the *cause of the change of the gradients from the normal to the abnormal, or convectional types*, is undoubtedly a direct contribution to the scientific theory of the local circulations of the air. The Barometry Report has furnished us for the first time with the data for constructing the isobars on the higher levels, as is shown on the charts published in the MONTHLY WEATHER REVIEW for January and February, 1903. The series of charts containing the daily isobars on three planes lays bare the mechanism of the dynamic structure in cyclones and anticyclones, so that it is necessary to develop a mathematical analysis in conformity with the observed facts. Furthermore, the same Barometry Report has furnished us with reliable annual residuals, or the variation of the pressure from year to year, and it has been shown in the MONTHLY WEATHER REVIEW for July, 1902,[1] that these residuals synchronize with similar pressure variations over the entire earth, and also to some extent with the annual variations in the frequency numbers of the solar prominences, faculæ, and sunspots. These facts suggest the foundation for the true cosmical meteorology of the future. These three lines of study—namely, (1) the abnormal system of convectional gradients, (2) the normal gradients when there are no vertical currents, and (3) the solar-terrestrial synchronous variations—can be developed with accuracy for the United States, and placed upon a strictly scientific basis. Could similar material be computed for several other parts of the earth, the whole cosmical research would be distinctly advanced from an empirical to a thoroughly scientific status. The point of view can be illustrated by employing the data obtained from the Cottage City waterspout, Table 51.

[1] Vol. XXX, pp. 347–354.

TABLE 51.—*Summary of the data for the Cottage City waterspout, August 19, 1896.*

Stages.		Metric system.					English system.					
		H. photo.	Height.	B.	t.	e.	H. photo.	Height.	B.	t.	e.	Gradient.
		mm.	Meters.	mm.	°C.	mm.	Inches.	Feet.	Inches.	°F.	Inches.	
δ-stage	Upper....	176.4	4942	414.5	− 12.0	1.64	6.95	16,214	16.32	10.4	0.065	
					Gradients.					Gradients.		
	Range....	73.6	2062	− 6.04 − 6.50	− .582 − .550	− .142 − .140	2.90	6,765	− .072 − .078	− .319 − .302	− .00170 − .00169	(G) Observed. (G) Cloud.
	Lower....	102.8	2880	539.0	0	4.57	4.05	9,449	21.22	32.0	0.180	
γ-stage		2.6	74	− 6.76	....	....	0.10	243	− .082	....	....	(G) Observed.
β-stage	Upper....	100.2	2806	544.0	0	4.57	3.95	9,206	21.42	32.0	0.180	
					Gradients.					Gradients.		
	Range....	61.7	1728	− 7.40 − 7.60 − 7.11	− .538 − .540 − .376	− .240 − .260 − .364	2.43	5,669	− .089 − .091 − .085	− .294 − .294 − .207	− .00288 − .00312 − .00437	(G) Observed. (G) Cloud. (G) Barometry.
	Lower....	38.5	1078	672.0	9.3	8.72	1.52	3,537	26.46	48.7	0.343	
α-stage	Upper....				Gradients.					Gradients.		
	Range....	38.5	1078	− 8.46 − 8.40 − 8.24	− 0.963 − 0.950 − 0.375	− 0.204 − 0.192 − 0.296	1.52	3,537	− 0.101 − 0.101 − 0.098	− 0.531 − 0.522 − 0.206	− 0.00246 − 0.00230 − 0.00355	(G) Observed. (G) Cloud. (G) Barometry.
	Sea level..	0	0	763.27	19.72	10.92	0	0	30.05	67.5	0.430	

(2) In the temperature of the α-stage the normal gradient is —0.206 F. per 100 feet, and the waterspout gradient is —0.531, which is only a little short of the true adiabatic gradient. The normal temperature fall from the sea level is —7.3°, or a change from 67.5° to 60.2°, at the 3537-foot level. This change from the normal temperature fall (which is small on the Atlantic coast in summer because of the southward bending of the isotherms over the ocean areas) to the adiabatic convectional gradient is a very striking fact. The latter gives a fall of —18.8° F., or a fall from 67.5° to 48.7° F., instead of to the normal temperature 60.2° F., *showing that something has occurred to suddenly reduce the normal temperature at the 3537-foot level by the amount 11.5° F.*

The cause of this will be explained in the following paragraph.

(3) The normal gradient of the vapor pressure is —0.00355 inch per 100 feet, for Nantucket in August, and this gives a total fall of —0.126 inch, or a change from 0.430 at sea level to 0.304 at the cloud base. The abnormal or convectional gradient prevailing at the waterspout is 0.00246, which makes a change of —0.087, or a diminution of the normal vapor pressure 0.430 to 0.343. Thus, we have a gain of 0.039 in the vapor tension for the waterspout. The result for the α-stage is a total *decrease* of the pressure by —0.110 inch, a total *decrease* in the temperature by 11.5° and a total *increase* in the vapor tension by 0.039 inch. This must be interpreted to mean that the air is rising from the sea level, carrying with it aqueous vapor into levels of temperature about —11.5° F. lower than that which prevails in the normal August weather.

(4) The depth of the α-stage is 3537 feet, of the β-stage 5669 feet, of the γ-stage 243 feet, and of the δ-stage 6765 feet, to the assumed top of the cumulo-nimbus cloud. In the β-stage the pressure gradient changes from the normal rate —0.085, to the convection rate —0.089 per 100 feet. This is equivalent to a fall of —4.82 inches in the normal state, from 26.58 to 21.76 inches; and to a fall of —5.04 in the convection cloud, from 26.46 to 21.42 inches. There is, therefore, a total fall in pressure of —0.227 inch, $(-.004 \times 56.7 = -0.227)$, induced by the change from the normal to the waterspout conditions in the β-stage of the cloud. We have thus a differential gradient per 100 feet in the α-stage of —0.0031, and in the β-stage of —0.0041 inch in favor of a vertical current. The excess in the β-stage over the α-stage of —0.0010 inch may be taken as the effective gradient due to the additional latent heat produced by the condensation of the aqueous vapor to liquid water. *This is only one-third the amount of the gradient due to the cause which produces the general uplift of the air that feeds the cloud.* This criterion also proves that there is a more efficient cause for the vertical pressure gradient than the condensation of the aqueous vapor, which has been so generally considered by meteorologists to be the true source of the energy that drives cyclones, following Espy's suggestion of fifty years ago.

(5) The temperature gradient in the β-stage changes, derived for the normal, is —0.207° F. per 100 feet, and —0.294° F. per 100 feet in the cloud. This amounts to a fall of —11.7° F. in 5667 for the normal state, and —16.7° F. for the convectional state, carrying the normal temperature from 60.2° to 48.5° F. in the β-stratum, and from 48.7° to 32.0° F. in the actual β-stage of the waterspout cloud. This is equivalent to a gradient excess in the α-stage of —0.325 of the convection over the normal gradient, and in the β-stage is one-fourth that in the α-stage. In the case of the pressure the excess in the β-stage is four-thirds that in the α-stage. The temperature difference of gradient diminishes rapidly in proportion to the height up to the γ-stage. In that stage and the δ-stage there is not much difference between the normal and the computed convectional gradients. This indicates that the effective cause of a vertical gradient is about exhausted at the height where the isotherms of the 0° is located, or, in other words that the *vertical convectional action is properly confined in the waterspout cloud to within about two miles of the ground, and is most active in the lower portion of the cloud.* In cyclones this vertical convection is usually limited to within two or three miles of the ground, though the accompanying dynamic action may penetrate into the upper strata as high as three or four miles; in hurricanes the penetration reaches to six or seven miles at least.

(6) In the β-stage the gradient of the vapor tension changes from the normal —0.00437 per 100 feet to —0.00288; the total fall in 5669 feet amounts to —0.248 inch, $(-0.00437 \times 56.7 = -0.248)$, in the normal, to —0.163 in the cloud convection, making the fall which should be from 0.304 to 0.056 in the normal state, from 0.343 to 0.180 in the cloud. The difference of gradient is +0.00109 per 100 feet in the α-stage, and + 0.00149 in the β-stage, showing that large quantities of vapor are carried upward from the sea level in both stages, but that there is a condensation of aqueous vapor to water equivalent to +0.00040 inch per 100 feet. We can not carry out this comparison between the normal and the convectional gradients in the γ-stage and the δ-stage, but the evidence is that they have become practically identical.

(7) It is desirable to compare these vertical gradients with the commonly observed horizontal gradients. We have in the pressure −0.34 inch in 9206 feet. This is equivalent to −13.47 inches in 364,525 feet, or 111,111 meters, or one degree in the standard latitude of forty-five degrees. Now, on the weather maps, −0.70 inch in five degrees, or −0.14 inch in one degree, is the average horizontal gradient in a highly developed cyclone. Hence, in the convectional cloud formation the vertical gradient is about one hundred times as large as in such horizontal motions. In the temperature the fall of −16.5° in 9206 feet is equivalent to −654° in 111,111 meters, or one degree, and this too is about one hundred times the horizontal gradients which are found on the weather maps. This indicates that *the scale of operations on the horizontal plane is only one hundredth that which occurs in the vertical direction in convectional clouds.* The linear dimensions of cyclones are usually about one hundred on the horizontal to one in the vertical direction, and these two facts, taken together, show how much less force is required to drive a horizontal than a vertical current.

THE CAUSE OF THE FORMATION OF THE WATERSPOUT CLOUD, AND THE VERTICAL CONVECTIONAL VELOCITY.

(1) *Vertical convection due to surface heating.*—We now reach the important question, what was the physical cause of the formation of the cloud, and the vertical convection within it that was the immediate condition of the generation of the vortex tube extending from the base to the ocean? Fortunately, one answer to this question is entirely excluded from our consideration. The disturbance of the normal stratification which produces an abnormal system of gradients and the corresponding vertical currents, may be due to two causes, (1) the surface layers may be overheated relatively to the upper layers, or (2) the upper layers may be undercooled relatively to the surface layers. Either cause would be equally efficient, and it is only a question of which one is actually operating in the case of this waterspout. Overheating the surface layers is due to a perfectly definite physical process, namely, as follows. The effective solar radiation falling upon the earth's atmosphere consists of short wave lengths from $0.30\,\mu$ to 2.00μ. (Compare figs. 3 and 4, Tables 1 and 2 of my paper on "Solar and terrestrial physical processes," MONTHLY WEATHER REVIEW, December, 1902, Vol. XXX, pp. 562–564.) These short waves, whatever may be the true effective solar temperature at which they are produced, penetrate to the surface of the earth with two sources of depletion, the first, by scattering in the upper atmosphere which cuts out a large percentage of them, and produces the strong glare that is characteristic of the higher layers; the second, by absorption in the aqueous vapor at certain wave lengths, which causes the observed depressions or cold bands in the energy spectrum. There is good reason for believing that the upper as well as the lower atmosphere is heated by the passage of the remainder of the short waves by only a very slight amount, and that this is practically negligible in general discussions. But these short rays falling upon the surface of the earth are readily absorbed, and this absorption powerfully raises the temperature of the land and ocean areas. That practically ends the history of the incoming solar radiation.

The terrestrial radiation, on the other hand, is of an entirely different character, and it has a very different effect upon the earth's atmosphere. The heat radiations at terrestrial temperatures, where the absolute temperature ranges from $T=200°$ to $T=325°$, have wave lengths extending from 2μ to 40μ, and, thus the outgoing wave lengths begin where the incoming lengths end. Many of these long waves, in radiating from the surface of the earth, are quite readily absorbed by the atmosphere, and the heat percolates from the lower through the upper strata by a process of slow conduction and convection. The aqueous vapor certainly absorbs many waves, as from 4μ to 8μ, and possibly most of the waves beyond 12μ. It was shown very distinctly in my International Cloud Report that the surface temperatures do not diminish with the height at an adiabatic rate, but much more slowly, as is indicated on charts 78 and 79. In Table 162 of the same report is given the number of calories per kilogram required to convert an adiabatic atmosphere into the actual atmosphere as observed. It shows an increase from the ground until the number is about 9.5 calories at the 13,000-meter level in summer and 11.0 calories in winter. This amount of heat may be taken to represent the effect of the outward flowing flux, which, like a slow conduction, keeps the upper atmosphere warmer than it would be if the outward-going waves had the same length as the inward-coming waves. This difference in wave length is the most important factor in the economy of the earth's atmospheric temperatures.

We may now fix our attention more closely upon the changes in the surface temperatures as measured in the normal diurnal and annual periods, and in the local variations of all kinds upon the average conditions. The change in the transparency of the atmosphere due to cloudiness, the difference of altitude of the sun, the character of the surface, whether water area, moist ground, or dry desert, all determine the effective temperature of the surface at any given time. These react upon the corresponding outgoing radiation, which first heats up the lowest strata of the atmosphere, or cools it, according to the prevailing conditions. Strong surface heating by day and cooling by night is, therefore, the regimen to which these layers are subject, and the integral effect of this action in its passage through the upper layers, finally builds up the observed normal gradients of temperature which by no means produce an abiabatic rate of stratification. Temporary convectional currents upward by day, downward by night, upward in some local areas, downward in other areas, constitute the common types of motion due to these causes. The formation of the lower cumulus clouds with moderate convection, of thunderstorms in strong convection, and of desert sand vortices are typical examples of purely surface overheating with vertical convection. But there is an entirely different class of vertical convection, to which sufficient attention has not been paid in meteorological investigations.

(2) *Vertical convection due to the overflow of cold upon warm currents of air.*—It is evident that the vertical convection in the cumulo-nimbus cloud of the Cottage City waterspout could by no possibility have been due to the overheating of the surface by the incoming solar radiation. The phenomenon occurred over the ocean and all the meteorological data of Table 50 show that there was actually no superheating effect near the surface at that time. The prevailing temperature was 67.5° F., while the normal for the month of August at Nantucket was 67.7° F.; and the humidity was 64 per cent, while the normal was 84.3 per cent. *In fact, the 19th of August, 1896, was the driest day of the entire month,* and the powerful vertical convection then taking place could not have been due to the solar radiation acting on the surface conditions. We must, therefore, look for another efficient principle capable of producing the powerful effects shown in the showers preceding the family of waterspouts and the thunderstorm with downfall of hail following them. This we can readily discover by referring to the weather chart of the date, August 19, 1896, fig. 37.

This map shows that a well-defined area of high pressure was just pushing its southeastern front over Vineyard Sound, that the winds were from the northwest, and that there was a fall in temperature of about 15° along the coast line, due to the advance of this cold area. We have, therefore, merely to assume, in accordance with the general fact, that the upper strata are moving eastward in advance of the lower, and that this cold air from the high area was blown forward over Vineyard Sound earlier in the strata a mile or two high than at the surface.

A sheet of cold air overran the low, warm, and quiet strata about midday, while the cold air followed at the surface a few hours later, and in these facts we have the exact conditions required to produce the observed powerful convection. Such abnormal cooling of the higher strata is as efficient in producing vertical convectional gradients as a superheating of the surface would be, and the evidence that this was the actual case is so good as to render it a practical proof of this circumstance. The upper strata were cooled suddenly by —12° to —15° F., and this brought showers, the waterspouts, and the thunderstorms in close succession. These were followed later by cooler conditions at the surface, giving a temperature fall from the maximum of the day, 72°, to the minimum, 56.5°, at Vineyard Haven. Under these conditions all the observed facts find so natural and satisfactory an explanation that no further remarks seem to be needed to enforce the theory.

But, it should be noted that this overflow of relatively cold layers of air at a moderate elevation upon the warm surface layers, this forereaching and temporary stratification causes an abnormal system of gradients which produce the vertical currents required to set up the motions that tend to reestablish the normal equilibrium of the atmosphere. This local disturbance of the average gradients, due to the fact that the cold upper air, under certain configurations of the lower currents, is drifted forward upon them, is the *primary cause of most of the phenomena classified as thunderstorms, tornadoes, cyclones, and hurricanes.* In short, all these violent local disturbances of the lower air are largely due to this cause, and this is the true source of the energy expended, though it has been attributed by one school of meterologists to the latent heat of condensation, and by the other school to the eddies established by differential horizontal velocities. These two latter sources of energy need not be excluded from consideration, for they contribute their quota to the total energy of circulation, but the first cause is the abnormal stratification of the air at moderate elevations. Thus the groups of thunderstorms which frequent the southeastern quadrants of the cyclone are due to the overflowing of the cold northern current upon the warm current from the south. Tornadoes have the same origin and their location shows that they are due to this cause. The cyclone itself is generated by warm currents of air from the Tropics underrunning the cold sheet which rotates above the surface of the earth, in the hemispherical whirl north of latitude 35°.

The reason for the outflow of warm currents from the Tropics has been indicated in the International Cloud Report, chapters 8 and 10; also there will be found in the MONTHLY WEATHER REVIEW for January and February, 1903, and in the preceding papers of this series, further illustrations and remarks on this theory. The West Indian hurricanes in a similar way are produced in the late summer and autumn by the overflow of the cool upper sheet from the North American Continent upon the warm tropical lower strata, because this sheet is then increasing in size with the southward retreat of the sun. The withdrawal of the sun to the south in fact brings the thermal equator of the higher strata toward the geographical equator earlier than that corresponding to the lower strata. Hence, relatively cold air from the temperate zones at considerable heights, begins to overlay the tropical warm and moist lower strata, and this induces the long continued vertical convection, localized in the hurricane vortex, which in its progressive movement may traverse thousands of miles along its parabolic track. The form of the track is due to the influence of the general circulation localized in centers of action, which builds the south Atlantic high area on the ocean and is manifested in the trade winds, so that the hurricanes usually gyrate along the edge of this special configuration.

The power which is expended for days in succession in a hurricane is due to the fact that the wide expanse of the upper cold sheet covers the temperate zones and overlaps the Tropics at moderate heights. As long as this contrast of temperature, due to abnormal stratification, continues, there is a sufficient source of energy in the resulting thermal engine to produce powerful vertical convection currents, and to sustain the most violent hurricanes, in which the vortex has a depth of several miles in a vertical direction. This theory seems to harmonize completely with what is known about the meteorology of the lower air, and to be such a satisfactory escape from the difficulties of (1) the condensation theory and (2) the dynamic eddy theory, which have always encountered both practical and theoretical objections, that we may expect to find confirmation of it in the future development of the mechanics of the atmosphere.

VARIATION IN TEMPERATURE OVER A LIMITED AREA.

By Prof. Willis I. Milham, Ph. D. Dated Williamstown, Mass., August 4, 1906.

I. INTRODUCTION.

The investigation of the variation in temperature over a limited area was continued and completed during the winter of 1905–6, and the present article contains the results of this investigation. The limited area in question is the village of Williamstown, Mass., which is about one and one-half miles long and three-quarters of a mile wide, and is situated on three small knolls in the middle of a larger depression. It consists of detached houses, surrounded by ample lawns and gardens, and has a diversified surface made up of fairly level areas, marked valleys, plenty of running water, and differences of elevation amounting to 120 feet.

One phase of the variation was investigated during the winter of 1904–5 and the results were published in the MONTHLY WEATHER REVIEW for July, 1905, Vol. XXXIII, page 305, under the title: "The variation in minimum temperatures on still, clear nights within the confines of a village". Here may also be found a complete description of the village, together with an accurate topographical map. A brief summary of that investigation will be useful as a preface for the present article. Accurate, self-registering, minimum thermometers of the regular Weather Bureau type were mounted in exactly the same way and exposed at ten different stations in the village. As long as the ground was covered with snow, their readings were noted whenever a decided fall in temperature occurred during the night. The maximum variation noted was 10° F. and the average variation for the thirty-six nights on which observations were taken was 5.1°. It was furthermore found that the variation was much less on a windy night, that it was not influenced by the coldness of the night, and that it was greatest on two different, carefully described types of nights. It was also found that air colder and thus more dense tended to drain into the valleys; but elevation was not the all-determining factor, for the openness of the valley, its direction, the roughness of its surface, and the wind direction also played a part. The regularity with which certain stations were either warmer or colder than other stations was investigated and it was found that one station proved to be quite constantly the coldest and another the warmest in the series of ten.

In order to determine completely the behavior of a limited area as regards variation in temperature, two more questions must be investigated. These questions are: (1) Does any variation in temperature exist during the daytime? (2) Does the variation remain constant during a given night, or does it change and perhaps grow larger as the time of minimum temperature approaches? It is the purpose of this article first to answer these two questions and then to summarize the results and give a complete picture of the behavior of a limited area as regards variation in temperature.